Chapter 8 exercises

1. Once again, consider an option with the payoff at exercise time t of

$$\max(0, K - S(t)^\alpha)$$

for an underlying asset $S(t)$, strike K and $\alpha > 0$. The stochastic dynamics are as discussed in the present chapter, i.e. S follows Black/Scholes–type dynamics, but with stochastic interest rates given by a Gauss/Markov HJM model. Implement code to do the following:

   (a) Price the European option with this payoff in closed form using the Skipper/Buchen formula, as generalised in this chapter.

   (b) Price the European option with this payoff by Monte Carlo simulation.

   (c) Apply the technique of multiple control variates used to generate Table 7.4 in the book to the Monte Carlo simulation of the European option in the present context.

   (d) Price the American option with this payoff by Monte Carlo simulation, using various choices of basis functions. Which choice of basis functions gives the best result? Does this depend on the number of training paths?

2. Consider a European payer swaption, i.e. an option to enter an interest rate swap as the payer of the fixed leg and receiver of the floating leg. In a Gauss/Markov HJM model with d factors, price this option by Monte Carlo simulation.

   (a) For $d = 1$, check the convergence to the closed form solution.

   (b) For $d = 2$, check the convergence to the solution given by numerical integration.

3. Repeat Exercise 1 for the following two payoffs, in the context of the multicurrency Gauss/Markov HJM model discussed in the present chapter, where $\tilde{S}$ is a foreign equity asset and X is the exchange rate:

   (a) Foreign asset struck in domestic currency:

$$\max(0, K - (X(t)\tilde{S}(t))^\alpha)$$

   (b) Quanto version of this option:

$$\max(0, K - (X(0)\tilde{S}(t))^\alpha)$$

4. A *Range Accrual Note* is an instrument which pays a given interest rate, but only based on the number of days a floating reference rate is within a given range. Thus a Range Accrual might pay interest of

$$\bar{r} \cdot 0.25 \cdot \frac{x}{y}$$

every three months, where $\bar{r}$ is the given interest rate, x is the number of days the reference rate (e.g. LIBOR) was within the given range $[r_{\text{lower}}, r_{\text{upper}}]$ during the three–month period, and y is the number of days in the three–month period. Suppose that the reference rate is the market interest rate for a time to maturity of three months, with simple compounding. Suppose the total time to maturity of the Range Accrual Note is three years and at maturity, the Note buyer receives from the Note issuer the final coupon payment and the notional ("face") value of the Note. Price this instrument by Monte Carlo simulation, with daily time steps,

   (a) where $\bar{r}$ is a fixed number,
   (b) where $\bar{r}$ is equal to the reference rate plus a fixed spread s, and
   (c) each of the above two cases, where the issuer has the option to redeem the Note early, by paying the buyer of the Note the notional value immediately after any coupon date.