Chapter 5 exercises

1. Implement code to price, in each of the three finite difference schemes discussed in this chapter, a European and an American option with the payoff at exercise time t of

$$\max(0, K - S(t)^{\alpha})$$

   for an underlying asset $S(t)$, strike K and $\alpha > 0$. Compare the results, in particular in terms of convergence, with the results of Exercise 1 in Chapter 3.

2. Adapt the finite difference scheme implementations in this chapter to price simultaneously several contingent payoffs in a single roll–back operation on the lattice.

3. Suppose two assets, S_1 and S_2, each follow Black/Scholes dynamics of the type

$$dS_i(t) = S_i(t)\left(rdt + \begin{pmatrix} \sigma_{i1} \\ \sigma_{i2} \end{pmatrix} dW(t)\right)$$

   with $W(t)$ a two–dimensional Wiener process (i.e. two–dimensional Brownian motion). Show how an option to exchange K units of S_1 for one unit of S_2 can be priced in a one–dimensional finite difference scheme of the types discussed in this chapter.